

CHAPTER NINE

MECHANICS OF SOLIDS

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9.1 INTRODUCTION

The mechanics that we have discussed so far (Chapters 3 to 8) is not based on any microscopic picture of matter. We have no doubt talked of 'particles'. A particle in mechanics is nothing but a body of negligible size with no internal structure. As we move on to the study of mechanics of solids and fluids, it is important to invoke the atomic constitution of matter.

All matter consists of extremely tiny bits called molecules. In elements like copper or sodium the molecules are simply atoms characteristic of the elements. The molecule of a chemical compound consists of a small number of atoms of different elements in close combination. For example, a molecule of carbon dioxide contains one atom of carbon and two atoms of oxygen. Sometimes, the tiniest bit of even an element is a molecule consisting of a few atoms of the element in chemical combination, as in gases like oxygen, nitrogen, etc. Polymers like nylon or bio-molecules like proteins have long chains consisting of hundreds or thousands of atoms.

What is the typical size of these atoms and molecules ? What holds them together and in what way are the three phases of matter (solids, liquids and gases) different ? We deal with some of these questions in the beginning of this Chapter. Later, in sections 9.6 and 9.7, we consider the macroscopic elastic properties of solids and their applications. The discussion there does not involve the atomic picture. The macroscopic elastic constants introduced therein, of course, arise from the detailed nature of interatomic and intermolecular forces. This macroscopic study of solids generally goes by the name of mechanics of solids.

9.2 MOLECULAR PICTURE OF MATTER

The ultimate building blocks of matter have been a subject of interest since antiquity. Is matter continuous all the way up to the minutest scale, or do we encounter discrete units

(atoms or molecules) as we go on sub-dividing matter? Questions like these were debated and scholars surmised the atomic picture of matter in the past long before it became part of modern science. Undermentioned boxes gives a glimpse of some of the brilliant insights of these ancient scholars.

element, the masses of the other element are in the ratio of small integers. Dalton's atomic hypothesis is : atoms are the smallest individual constituents of matter; atoms of an element are identical and differ from those of other elements in mass. When elements combine to form a compound a small

Atomic Hypothesis in Ancient India and Greece

Though John Dalton is credited with the introduction of atomic viewpoint in modern science, scholars in ancient India and Greece conjectured long before the existence of atoms and molecules. In the Vaisesika school of thought in India founded by Kanada (Sixth century B.C.) the atomic picture was developed in considerable detail. Atoms were thought to be eternal, indivisible, infinitesimal and ultimate parts of matter. It was argued that if matter could be subdivided without an end, there would be no difference between a mustard seed and the Meru mountain. The four kinds of atoms (**Paramanu** – Sanskrit word for the smallest particle) postulated were Bhoomi (Earth), Ap (water), Tejas (fire) and Vayu (air) that have characteristic mass and other attributes, were propounded. Akasa (space) was thought to have no atomic structure and was continuous and inert. Atoms combine to form different molecules (e.g. two atoms combine to form a diatomic molecule *dvyanuka*, three atoms form a *tryanuka* or a triatomic molecule), their properties depending upon the nature and ratio of the constituent atoms. The size of the atoms was also estimated, by conjecture or by methods that are not known to us. The estimates vary. In *Lalitavistara*, a famous biography of the Buddha written mainly in the second century B.C., the estimate is close to the modern estimate of atomic size, of the order of 10^{-10} m.

In ancient Greece, Democritus (Fourth century B.C.) is best known for his atomic hypothesis. The word 'atom' means 'indivisible' in Greek. According to him, atoms differ from each other physically, in shape, size and other properties and this resulted in the different properties of the substances formed by their combination. The atoms of water were smooth and round and unable to 'hook' on to each other, which is why liquid /water flows easily. The atoms of earth were rough and jagged, so they held together to form hard substances. The atoms of fire were thorny which is why it caused painful burns. These fascinating ideas, despite their ingenuity, could not evolve much further, perhaps because they were intuitive conjectures and speculations not tested and modified by quantitative experiments - the hallmark of modern science.

The earliest scientific evidence for atoms came from chemistry. John Dalton, an English chemist, suggested that the empirical laws of chemical combination find a natural explanation in terms of the atomic hypothesis. The **law of definite proportions** says that any given compound has a fixed proportion by mass of its constituent elements. The **law of multiple proportions** refers to the case when two elements combine to form different compounds. It says that for a fixed mass of one

number of atoms of each of the elements combine to form the molecule of the compound. This picture combined with Avogadro's hypothesis (**equal volumes of all gases at any given temperature and pressure contain equal number of molecules**) also provided a simple explanation of Gay Lussac's law: **when gases combine chemically to yield another gas, their volumes are in the ratio of small integers**. You will be familiar with these matters from your chemistry courses.

Modern Atomic View of Matter : Early Pioneers



John Dalton
(1766 – 1844)

English chemist, who postulated the atomic theory of matter, based on empirical laws of chemical combination (law of constant proportions and law of multiple proportions). Chemistry, indeed all of science, was thus put on a firm foundation. He is also known for Dalton's law of partial pressures for a mixture of gases and for his contributions to meteorology, thermal expansion of gases and theory of colour blindness.

Italian physicist whose hypothesis known by his name provided a simple molecular explanation of why gases combine in simple proportions by volume. He also suggested that the smallest constituents of gases like hydrogen, oxygen and nitrogen are not atoms but diatomic molecules. Avogadro's number



Amedeo Avogadro
(1776 – 1856)

(6.023×10^{23}) is an important constant of nature and establishes the scale of atomic masses.

A significant support to the molecular picture of matter came from the **Kinetic theory of gases** that we discuss in Chapter 11. This theory gave successful molecular interpretation to various properties of gases and also yielded estimates of molecular sizes. Despite this evidence, as late as the end of the nineteenth century, some scientists were still not convinced of the existence of atoms and molecules and regarded them as fictitious objects introduced for convenience of theoretical analysis. A more direct confirmation of the reality of atoms and molecules came from the phenomenon of Brownian motion (Chapter 11).

From a large body of evidence, we now know that the size of the atom is typically in the range of angstroms ($1 \text{ \AA} = 10^{-10} \text{ m}$) and its mass is given on the scale of atomic mass units ($1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$). Avogadro's number—the number of atoms or molecules in the atomic or molecular mass of the substance in grams—is 6.023×10^{23} . One mole of a substance is defined to be the amount of substance in which the number of molecules is equal to the Avogadro's number. Ordinary matter like a page of this book, the cooking gas in a cylinder or a pail of water thus have an enormously large number of molecules.

With the advent of high magnifying and resolving power instruments like electron microscopes and scanning tunneling microscopes we can now 'see' the arrangement of atoms or molecules as they actually are in a solid. We can also get an idea about their size and measure the separation between them in a typical solid. The inter-atomic distances in solids are of the order of a few angstroms. For liquids also, the interatomic distance is of the same order, since typical densities of solids and liquids are not very different. In gases on the other hand, the interatomic distance is about ten times that in a solid or liquid.

9.3 INTERATOMIC AND INTERMOLECULAR FORCES

What holds the atoms in a solid or liquid? This is the question we now consider. Evidently there must exist some interatomic or intermolecular forces that keep them bound. However, the detailed nature of these forces is rather complex. In this section we shall, therefore, present only a qualitative description of these forces and see how

various physical properties of matter can be understood in their terms.

We know that solids are characterised by their shape and size; large deforming forces are required to bring about any change in their shape and size. Similarly liquids have definite size or volume but not shape, they can easily flow. The gases, on the other hand, do not have any shape or size. These observational facts suggest that the interatomic or intermolecular forces are perhaps strongest in the solids and weakest in the gases. These forces are attractive and depend on the interatomic or intermolecular separation. But if they are always attractive and their strength increases with decreasing separation, the matter should collapse when the atoms are brought closer and closer. This is contrary to observation; it is also known that the solids are highly incompressible. These suggest that the nature of interatomic forces changes when the atoms are brought too close, they become repulsive rather than attractive.

The interatomic forces are electrical in nature. As you know atoms consist of very small positively charged nuclei (size $\sim 10^{-15} \text{ m}$) and negatively charged electrons distributed over a distance of $\sim 10^{-10} \text{ m}$ (atomic size). Each atom is electrically neutral. Now if they are electrically neutral, how do they attract each other? Strictly speaking atoms and molecules are not hard spheres, they do not have sharp boundaries. An atom has around the nucleus, distribution of negative (electron) charge, like a cloud. The atomic size is the 'size' of this cloud or distribution. Each atom or molecule undergoes fluctuating distortions in shape. Such a distortion produces a relative shift between the centres of positive and negative charges in each atom. Thus it behaves like an oscillating electrical dipole having a dipole moment. It can be shown that two oscillating dipoles always attract each other. The interaction energy between them varies as R^{-6} . This leads to an attractive interaction between two atoms (or molecules), called van der Waals attraction in the honour of the Dutch physicist C.F. van der Waals who postulated such an attractive force from his study of liquefaction of gases. Detailed calculations as well as deductions from experiment show that the interaction between any isolated pair of atoms or molecules may be represented by a curve

that shows how the potential energy varies with separation between them, as in Fig. 9.1. This curve describes the interatomic potential.

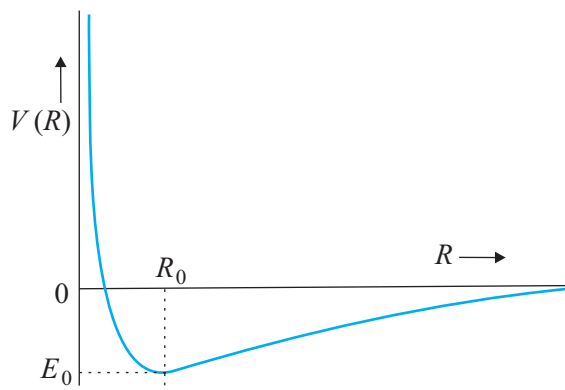


Fig. 9.1 Interatomic potential energy $V(R)$ between two identical atoms as a function of separation R between their nuclei

The force between the atoms can be found from the potential energy by using the relation,

$$F(R) = -\frac{dV}{dR}$$

The resulting interatomic force curve is shown in Fig. 9.2.

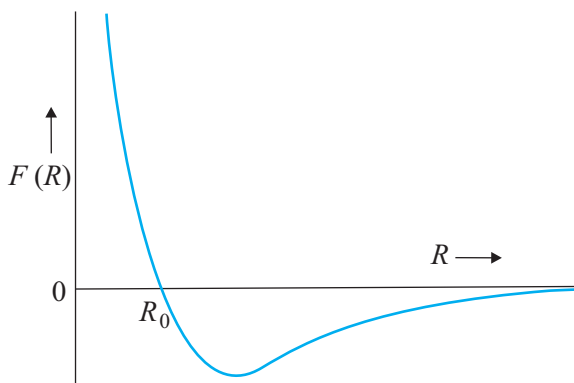


Fig. 9.2 Interatomic force $F(R)$ between two identical atoms as a function of separation R between their nuclei

The force is along the line joining the two atoms or molecules, and is shown negative for attraction and positive for repulsion. We see that as the distance R decreases, the attractive force first increases and then decreases to zero at a separation R_0 where the potential energy is minimum. For smaller distances the force is repulsive, because at these distances the

negative charge distribution associated with one atom begins to overlap with that associated with the neighbouring atom. The general form of the potential energy curve can be represented by

$$U(R) = \frac{A}{R^n} - \frac{B}{R^m} \quad (9.1)$$

The coefficients A and B and the exponents (n and m) depend on the nature of atoms (or molecules) in question. The first term represents the repulsive part of the potential and the second represents the attractive part. The exponents n and m , for most substances, are 12 and 6, respectively. Thus the repulsive part of the potential has a very short range and becomes effective only when atoms are brought very close to each other.

The above picture of interatomic or intermolecular forces is an over simplification of the actual situation. However, it provides a reasonable approximation.

9.4 STATES OF MATTER

Matter exists in three states, namely gas, liquid and solid. A fourth state of matter consisting of ionised matter is called **plasma**. However, in our further discussion we will restrict only to the first three states of matter.

The existence of three states of matter, namely gas, liquid and solid is an important consequence of the interatomic or intermolecular forces. A collection of the same atoms or molecules (e.g. Na or N_2) can be in one or the other of the three states, depending on conditions such as temperature and pressure. A gas has the volume of the closed container in which it is kept; a liquid has a fixed volume at a given temperature but no shape; a solid has both volume and shape. What causes them to behave differently? These are mainly an interplay of two factors, (a) interatomic or intermolecular forces and (b) the agitation or random motion due to temperature.

In a gas, the atoms or molecules are far apart. The average spacing between molecules is about ten times their diameter. The forces between them are weak. The molecules are constantly moving about; their kinetic energy is proportional to the temperature (see Chapter 11). As the forces are weak, they move about essentially as free particles, colliding against each other occasionally. We shall discuss this picture of a gas in greater detail later.

Now suppose the gas is cooled, the molecules slow down, and (or) the pressure is increased so that the average distance between the molecules decreases. The molecules are now more likely to be at distances where the intermolecular potential is sizeable and attractive. If the average kinetic energy of a molecule is less than the average attractive potential energy, it condenses into a liquid. If the kinetic energy is larger the molecules fly away, and the system is a gas. Thus on cooling and compressing a gas, a situation is reached when the gas condenses into a liquid.

In a liquid the molecules are close to each other. In **cold liquids** (i.e. at temperatures well below the boiling point) the typical intermolecular separation is close to but little larger than R_0 , the separation for which the potential energy has a minimum. However, in a liquid the molecules still move in a chaotic manner, though much more slowly as compared to a gas. Liquids cannot stand any deformation; on applying a force tangential to the surface, they begin to flow.

Suppose the liquid is cooled further, the molecules come still closer. A stage will be reached when the intermolecular separation is close to R_0 . The molecules will then tend to adopt a configuration of minimum energy, i.e. an arrangement of molecules such that each one of them is at a distance close to R_0 from its nearest neighbour and the nearest neighbours are arranged symmetrically so that each molecule is in a position of minimum potential energy. This regular or near regular arrangement is a solid. The molecules are still in motion, but the motion of each molecule is around a fixed mean position. The molecules or atoms execute vibratory motion about their fixed mean positions. The amplitude of motion depends on temperature but is small.

If the molecules have a shape very different from a sphere (e.g. they are like rods or discs) other kind of phases are possible. For example, a collection of rod like molecules can condense into a 'liquid crystalline' phase that is different from an isotropic liquid and a crystalline solid.

The macroscopic properties of matter in three different states discussed above are quite different. In the present chapter we shall restrict ourselves to the mechanical behaviour or mechanics of solids. The mechanics of fluids

in general will be dealt with in the next Chapter.

9.5 SOLIDS

When we think of a rigid body, the picture, which comes to our mind is that of a hard object having definite shape and size, a **solid**. Inside a solid, each atom or molecule is at a fixed average separation with respect to other atoms or molecules. However, by virtue of thermal energy, the atoms or molecules vibrate about their mean positions. The average separation between the atoms or molecules does not change with time. The location of each atom or molecule is fixed by the requirement that every atom or molecule be in equilibrium i.e. there is no net force on any one of them due to the others. The equilibrium is stable; if an atom or molecule is displaced from its equilibrium position, a restoring force, tending to push it back, acts on it due to other atoms or molecules. Because of this property, a solid is rigid within limits. It is stable against forces which tend to change the relative distance between the atoms or molecules and consequently the shape or size of the solid. This is the cause of elastic behaviour of solids. However, if the force is too large it may cause permanent deformation, the solid may break or flow.

The spatial arrangement of atoms or molecules in a solid may vary. In some solids they are arranged in a regular three-dimensional periodic array whereas in others the arrangement may be completely random. The solids we come across in our daily life could broadly be classified in three groups : crystalline solids, semi-crystalline solids, and glassy or amorphous solids.

In crystalline solids such as sugar, common salt, diamond etc., atoms or group of atoms are arranged in a perfectly periodic arrangement exhibiting translational symmetry in three dimensions. In such solids, if we move in any direction, after a definite interval the arrangement is repeated. Any lack of such symmetry leads to **glassy or amorphous solids**, examples of which can be found in materials like glass, bitumen, wood etc. Polymers are the building materials of modern times. Polymer molecules are very large; they are characterised by a large number ($\sim 10^{10}$) of basic repeat units. Polyethylene is perhaps the simplest polymer and finds extensive use in our daily life. The

buckets and carry-bags we use are made of this material. The polyethylene molecule is represented by $(-\text{CH}_2-)_n$, where n is the number of repeat units. These molecules are long chain molecules having very high molecular masses and are therefore termed as macromolecules. Protein molecules also fall under this category. In case of such materials a single molecule may run in a large volume. When such molecules are cooled from the liquid phase or the molten state (melt), they acquire a configuration as shown in Fig. 9.3. Here we find that there are some regions where the molecular chain is arranged in a regular manner. These regions are called crystallites. In between these crystallites the chain is folded in an irregular manner. Such regions constitute the amorphous regions. Thus we find that in these materials the crystalline phase is inter-dispersed in the amorphous phase. Such materials are called semi-crystalline solids. In this Chapter we discuss only crystalline and amorphous solids.



Fig. 9.3 Chain folding in a macromolecule. There are regions where the chain is folded in a regular manner (marked by dotted encircles). Such regions constitute the crystalline regions called crystallites. Portions where the chain is folded in a random manner constitute the amorphous region

All solids are to some extent **elastic**. We can change their dimensions slightly by pulling, pushing, twisting or compressing them. In the later parts of this Chapter you will learn about the elastic behaviour of solids under the influence of various types of deformations.

9.5.1 Crystalline Solids

It has been known and described for several thousand years that many minerals and gems have regular external form. A diamond, small or large, derived from any source has the same external form. The word crystal referred first

only to ice and then to quartz until the late middle ages when the word acquired a more general meaning. Any solid, which exhibited regularity of external form, was classified as a crystalline solid.

The regularity of appearance and external form of the crystals found in nature (see Fig. 9.4) or grown in the laboratory led observers, since the seventeenth century, to the belief that crystals are formed by a regular repetition of identical building blocks. In Fig. 9.5, it is seen that repetition of similar building blocks can lead to different external forms of crystals. When a crystal grows in a constant environment, the shape remains unchanged during growth, as if identical elementary building blocks are added continuously to the crystal. We now know that the elementary building blocks are atoms or group of atoms; crystals are a three-dimensional periodic array of atoms (see Fig. 9.6). The periodic arrangement is repeated regularly over large distances and thus the system exhibits a long-range order. Such a repetitive three-dimensional arrangement is called a lattice. In a lattice, each atom has a well-defined equilibrium distance from its nearest neighbours. Inter-atomic forces hold the atoms together; they may be visualised as tiny springs interconnecting the neighbouring atoms. The lattice is remarkably rigid, which is another way of saying that the 'inter-atomic springs' are extremely stiff. This fact is responsible for the definite shape and size of solids.

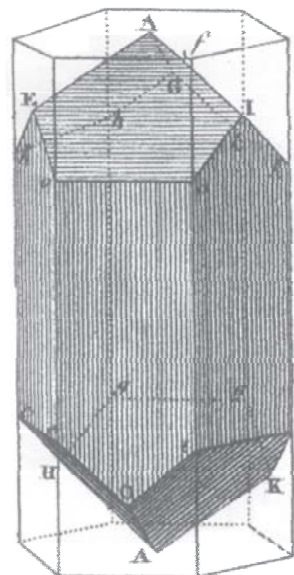


Fig. 9.4 Sketch of a quartz crystal

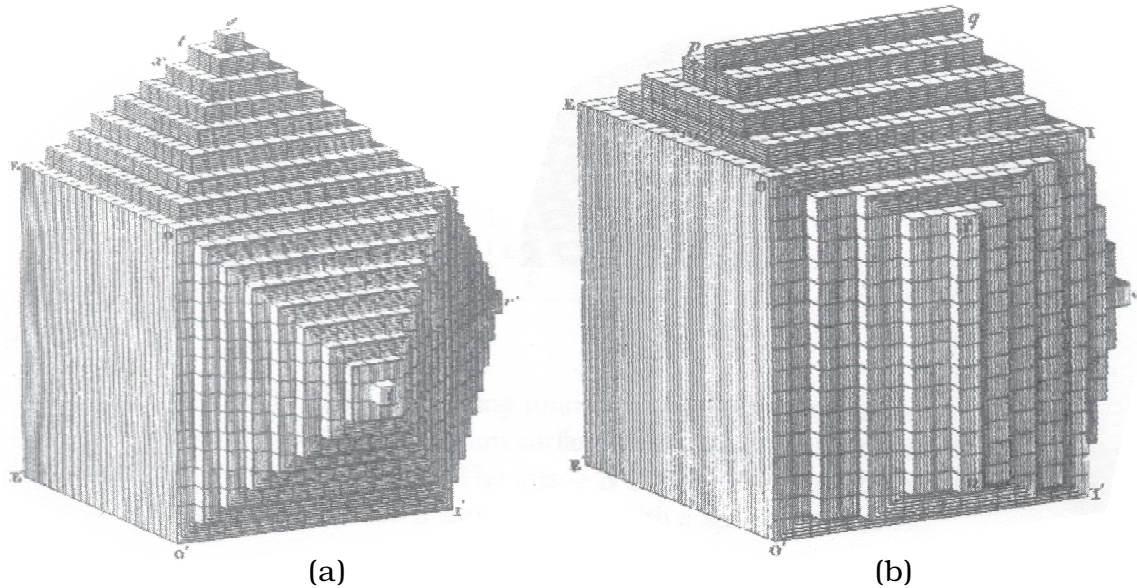


Fig. 9.5 Relation of the external form of crystals to the form of the elementary building blocks. The building blocks are identical in (a) and (b), but different crystal faces developed

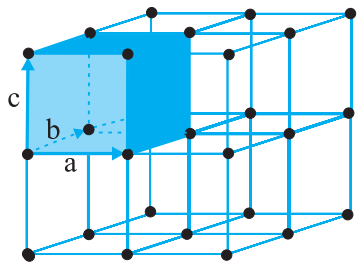


Fig. 9.6 A three-dimensional array of atoms, the shaded cube constitutes the repeat unit. Such a structure is called the lattice

How do we know that atoms or group of atoms in a crystal are arranged in a three-dimensional lattice? A fact, which strongly suggests this, is the presence of flat faces in naturally and artificially grown crystals. However, there are many direct ways of exploring the arrangement of atoms in a crystal. Extremely high resolution and magnification by electron microscopes and scanning tunnelling microscopes (STM) (see Fig. 9.7) has made it possible to 'see' directly the arrangement of atoms in a crystal lattice. The most common method is X-ray diffraction. X-rays are electromagnetic waves with a very short wavelength $\sim 1\text{--}10 \text{ \AA}$. The atoms of the crystal diffract these waves. Because of the regular arrangement of atoms in a crystal it turns out that X-rays diffract most intensely in some special directions, which depend on the

arrangement of the atoms. From detailed studies of these directions we can not only find the structure of simple crystals like copper and rock salt but also of complex crystals of DNA.

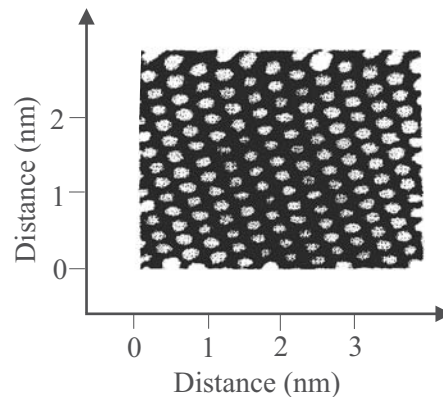


Fig. 9.7 A scanning tunnelling microscope image of an aluminium surface. Each bright spot represents a single atom. The image is slightly distorted: the atoms lie on a square lattice with a spacing of 0.2 nm

9.5.2 Glassy or Amorphous Solids

Many solids like glass, bone, wood etc. are not crystalline. Atoms in these materials are not arranged in a regular array but in a random manner as in a liquid. But unlike liquids where the molecules move relative to each other, here the atomic or molecular locations are fixed with respect to one another. As a consequence, these solids like the crystalline solids also possess

definite shape and size. Such substances are called glassy or amorphous solids. A projection in two dimensions of the atomic arrangement in such solids can be represented as shown in Fig. 9.8. Here for comparison, the corresponding arrangement in a two dimensional crystalline solid is also shown.

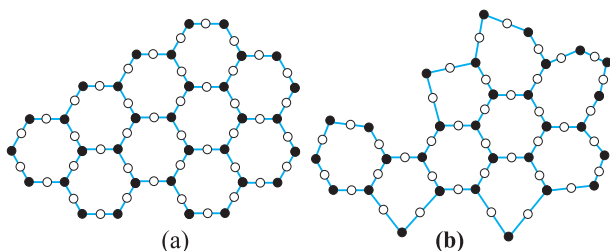


Fig. 9.8 This figure illustrates the difference between a crystal and a glass. In (a) Si and O atoms are imagined as forming a hypothetical two-dimensional crystal. In the glass (b), the long-range order of the crystal is destroyed without greatly distorting the bonds on any one atom

A surprising fact is that many substances forming glasses (for example quartz, i.e. silica) also have a crystalline phase (of lower energy). How is it that a less stable arrangement continues for a long time (thousands of years) and a more stable crystalline arrangement is often not found? To understand this let us consider the cooling curve of a substance from its liquid or melt phase as shown in Fig. 9.9. In this figure it is shown that when a liquid or melt is cooled slowly, it follows the curve 1-2-3-4. On the other hand if it is cooled rapidly, the curve 1-2-5-6 is followed. In the first case, at a temperature T_m there is an abrupt change in specific volume (volume per unit mass) without any appreciable change in temperature. Such a change represents a phase transformation. At this temperature, the liquid solidifies into a crystalline solid. This temperature is called the melting or crystallisation (solidification) temperature. On cooling rapidly, even at temperatures lower than T_m the liquid does not solidify but continues to remain in the liquid phase. The liquid is then said to be in a supercooled state, such as in region 2-5 of the cooling curve. On further cooling, lower than a temperature T_g , it transforms into a **glassy** or **amorphous solid**. The temperature T_g , called the glass transition temperature, is

characteristic of the material. Thus for most of the solids the crystalline state is the natural one since the energy of the ordered atomic arrangement is lower than that of an irregular packing and a state with lower energy is more stable. However, when the atoms or group of atoms are constrained or not given an opportunity to arrange themselves properly, by inhibiting their mobility, amorphous materials may be formed. Amorphous carbon is formed as a decomposition product at low temperatures.

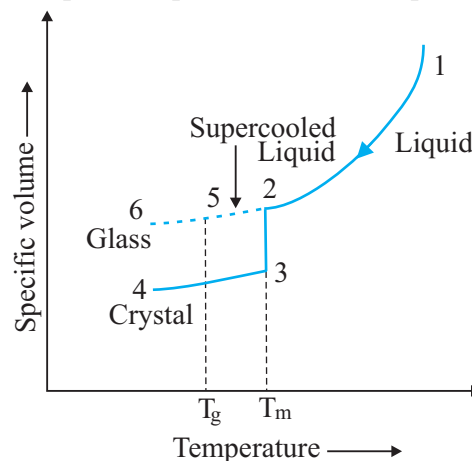


Fig. 9.9 The cooling curve of a substance from its melt phase. On slow cooling, it follows the path (1-2-3-4); at temperature T_m the material solidifies into a crystalline solid. On rapid cooling it follows the path (1-2-5-6); it continues to remain in the melt phase even below T_m . It solidifies into an amorphous solid at temperatures below T_g .

9.6 ELASTICITY : STRESS AND STRAIN

It was pointed out earlier that a solid has a **shape** and **size**. In order to change or deform the shape or size, a force is required and usually it has to be maintained if the solid is to be kept in its new shape or size. If you stretch a spring, the force needed to keep the spring stretched to the same extent does not change with time. Consider a small section of the spring close to the end where an external force is applied. Since it is in equilibrium, the net force on this section is zero. The rest of the spring is exerting an equal and opposite force. This force is called the restoring force, because it is acting in the direction needed to restore the spring to its original length. A solid, exhibiting this behaviour of exerting a restoring force, when deformed, is called an **elastic** solid. If the solid returns exactly to its original shape

when the external force is removed, it is said to be **perfectly elastic**. However, no solid is perfectly elastic, it behaves so only when the deformation produced is small. To get a feel for the orders of magnitude involved, consider a steel rod of length 1 m and diameter 1 cm. If you hang a medium size car (mass ~ 3000 kg) from the end of such a rod, the rod will stretch, but only $\sim 0.05\%$. Furthermore the rod will return to its original length when the car is removed. If you hang two cars from the rod, the rod will be permanently stretched and will not recover on removing the cars. If you hang three cars from the end, the rod will break. The elongation of the rod just before breaking is $\sim 0.2\%$. Although the deformations of this magnitude seem to be small, but are important from engineering point of view.

Robert Hooke, an English physicist (1635-1703), while experimenting with springs, found that in any springy body, the extension is proportional to the load. The relation between load and extension is, like Boyle's law, one of the earliest quantitative relations in science. It is very important to know the behaviour of materials under different kinds of loads. This information is very vital for designing buildings and bridges. Similarly, one can ask the question—can we design an aeroplane, which is very light but sufficiently strong? Why do the rails (on which railway trains run) have a particular shape like that of **I**? Why is glass brittle and not brass? Answers to such questions begin with the study of how relatively

simple kinds of loads or forces act to stretch, compress or deform different solid bodies.

As said above, when an external force deforms a solid, internal restoring forces, opposing the change, are developed in the body giving rise to **stress**. There are three ways in which a solid might change its dimensions when external forces act on it. These are shown in Figs. 9.10. In Fig. 9.10(a), a cylinder is stretched by two equal forces applied normal to its cross-sectional area. In Fig. 9.10(b), two equal and opposite forces, applied parallel to its cross-sectional area and perpendicular to its axis, tend to deform the cylinder much like a pack of cards. In Fig. 9.10(c), a solid object placed in fluid under high pressure is compressed uniformly on all sides. In all deformations, a change in the relevant dimension per unit dimension termed as **strain** is produced due to the deforming force resulting in a **stress** (defined as restoring force per unit area). In Figs. 9.10 **tensile stress** (associated with stretching) is illustrated in (a), **shearing (tangential) stress** in (b), and **compressional or hydraulic stress** in (c). The stresses and strains take different forms in the three situations depicted in Fig. 9.10. Empirically the stress and strain are proportional to each other. Thus,

$$\begin{aligned} \text{stress} &\propto \text{strain} \\ &= k \times \text{strain} \end{aligned} \quad (9.2)$$

The proportionality of Eq. (9.2) is valid only for small deformations. This is known as Hooke's

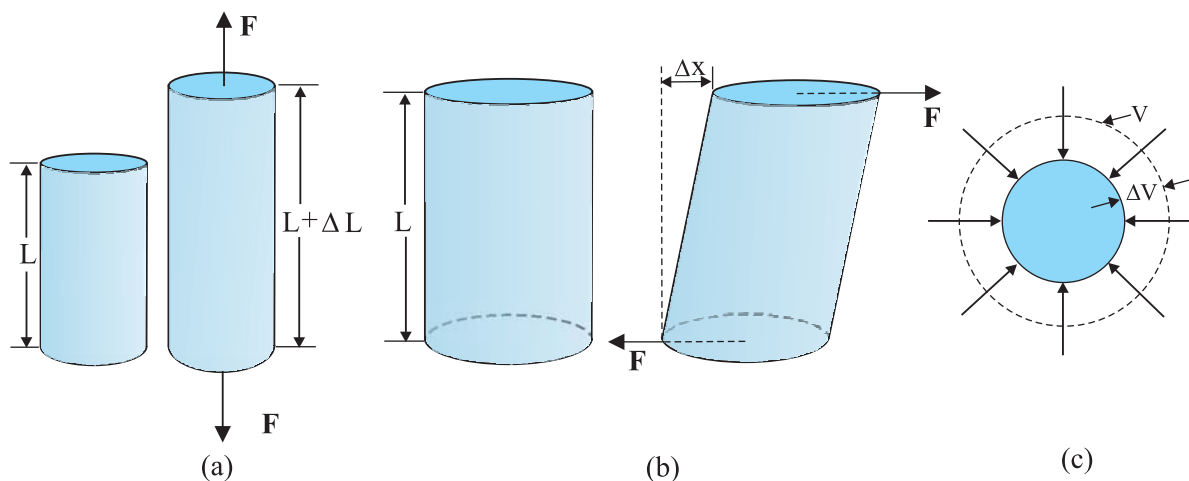


Fig. 9.10 (a) A cylinder subject to tensile stress stretches by an amount ΔL . (b) A cylinder subject to shearing (tangential) stress deforms by an amount Δx , somewhat like a pack of cards would. (c) A solid sphere subject to uniform hydraulic stress from a fluid shrinks in volume by an amount ΔV

law. The constant of proportionality k is called **modulus of elasticity**.

In a standard test of tensile properties, the tensile stress on a test cylinder, like the one shown in Fig. 9.11, is slowly increased from zero to the point at which the cylinder fractures,

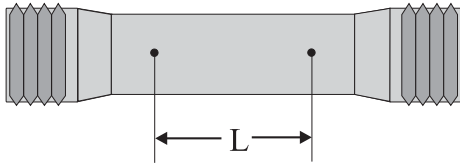


Fig. 9.11 A test specimen, used to determine a stress-strain curve. The change ΔL , that occurs in a certain length L is measured in a tensile stress-strain test

and the strain is carefully measured. A graph is plotted between the stress and the strain produced. Such a graph is shown in Fig. 9.12. It is observed that for a substantial range of stresses, the stress-strain relation is linear until the point A, and the specimen recovers to its original dimensions when the load is removed. In this region the solid behaves as a perfectly elastic body and it is here that the Eq. (9.2) applies.

From A to B, stress and strain are not proportional, but nevertheless, if the load is removed at any point between O and B, the curve will retrace and the material will return

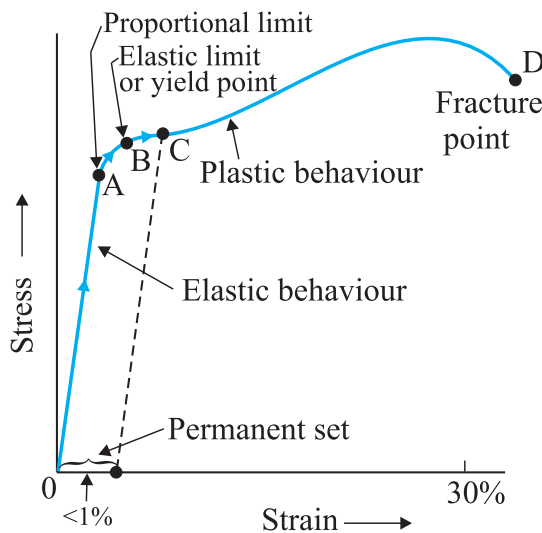


Fig. 9.12 A typical stress-strain curve

to its original length. In the region between O and B, the material is said to be **elastic** or to exhibit **elastic behaviour**, and the point B is called the **elastic limit** or the **yield point**. The stress corresponding to the point B is called the **yield strength** S_y . Up to this point, the forces exerted by the material are **conservative**; when the material returns to its original shape the work done in producing the deformation is recovered. If the stress is increased beyond the yield strength S_y of the specimen, the strain increases rapidly but when the load is removed at some point beyond B say C, the material does not come back to its original length but traverses the dashed line in Fig. 9.12. The length at zero stress is now greater than its original length and the material is said to have a **permanent set**. Thus on reducing the stress to zero now one does not get back to the unstrained state, but some strain remains. The fact that the stress-strain curve is not retraced on reversing the strain is known as **elastic hysteresis**. Further increase of load beyond point C produces a large increase in strain until a point D is reached at which **fracture** takes place. In the region B to D, known as the **plastic region**, the material is said to undergo **plastic flow** or **plastic deformation**. If large plastic deformation takes place between the elastic limit and the fracture point, the material is said to be **ductile**. If, however, the fracture occurs soon after the elastic limit, the material is said to be **brittle**. The stress at which the specimen eventually ruptures is called the **ultimate strength** S_u or the **tensile strength** of the material.

In nature there are materials which behave quite differently from what has been described above. Rubber, we know, can be pulled to several times its length and still returns to its original shape. Also, there is no well-defined plastic flow region; rubber just breaks when stretched beyond a certain limit. In Fig. 9.13, the stress-strain curve for a rubber like material, namely the elastic tissue of the aorta is shown. Notice that while the elastic region is very large, the material does not obey Hooke's law at all. Substances, which can be stretched to large values of strain, are called **elastomers**. In our body, the elastic tissue of aorta (the large vessel carrying blood from the heart) is an elastomer.

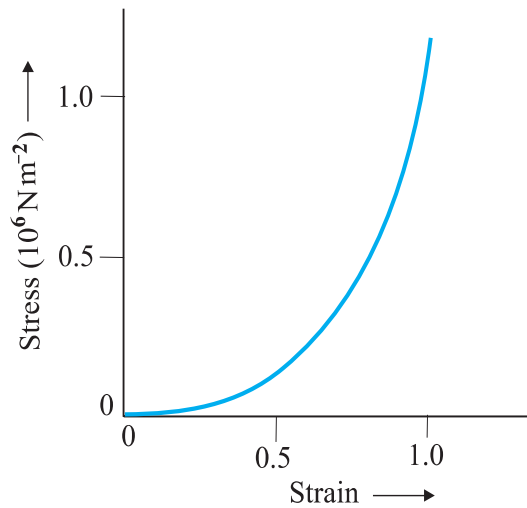


Fig. 9.13 Stress vs. strain for the elastic tissue of the aorta, the large tube (vessel) carrying blood from the heart

We will now discuss various kinds of deformations mentioned above.

9.6.1 Tension and Compression

When a cylinder or a wire is stretched, by applying two equal and opposite forces at the two ends but directed away from each other as shown in Fig. 9.10(a), it is said to be under tension or subjected to a tensile stress. Similarly, if two equal and opposite forces are applied at the two ends of a cylinder and are directed towards each other, the cylinder is said to be under compression.

A typical experimental arrangement to study the extension of a body under tension is shown in Fig. 9.14. Here a long straight wire of uniform cross-section is suspended from a fixed support. At the other end there is a pan in which known weights can be placed. The weights placed in the pan exert a downward force while an equal and opposite reaction acts at the support. The wire is thus stretched under stress. The extension of the wire is measured by a vernier arrangement where the vernier scale is attached to a pointer at the bottom of the wire, and the main scale is fixed to a stand or wall. Results of the experiment show that the elongation

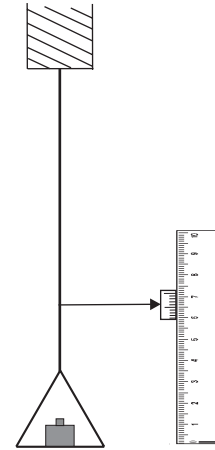


Fig. 9.14 An arrangement for studying extension of a wire

(increase in length) for a given weight is proportional to the length of the wire and depends inversely on the area of cross-section. The elongation for a given wire is proportional to the weight placed in the pan. From such results, the English physicist Thomas Young argued in 1807 that the load and extension are most naturally defined by quantities called stress and strain. **Stress is equal to the force (weight) per unit area acting normal to its cross-section.** If the weight pulling the wire is W , and this acts on the cross-sectional area A of the wire, the stress σ is

$$\sigma = \frac{W}{A} \quad (9.3)$$

and has the dimensions of force per unit area (with SI unit N m^{-2}). The extension depends on stress σ , and not on W or A . Similarly, for a given stress, the elongation ΔL is proportional to the original length L . Thus we define a dimensionless quantity called strain, ϵ , given as

$$\epsilon = \frac{\Delta L}{L} \quad (9.4)$$

Strain can be measured conveniently by a strain gauge*.

Young suggested that Hooke's law is described as $\epsilon \propto \sigma$, or as

$$\sigma \propto \epsilon \quad (9.5)$$

* It is a simple and useful electrical device, which can be attached to the operating machinery with an adhesive. It operates on the principle that its electrical resistance depends on the strain it undergoes

i.e. stress is proportional to strain. The constant of proportionality Y is called the Young's modulus and Eq. (9.5) can then be written as

$$\sigma = Y \varepsilon \quad (9.6)$$

Young's modulus is also called as the modulus for tensile or linear compressive stress. Dimensionally, the Young's modulus is a force per unit area and therefore has the unit N m^{-2} and is measured in pascals (Pa). Although the Young's modulus for an object may be almost the same for tension and linear compression, the object's ultimate strength may well be different for the two types of stress. Concrete for example is very strong in compression but is so weak in tension that it is never used in that manner.

Elastic properties of some materials of common interest are given in Table 9.1. Note that for metals the Young's modulus is very large ; therefore, in these materials it requires large forces to produce small changes in length. To increase the length of a thin, 0.1 cm^2 cross-section, copper wire by 0.1% requires a force of $\sim 10^3 \text{ N}$. Also most metals have Young's modulus in the range 10^{11} N m^{-2} . Bone, which is a mixture of several materials, has a rather small Young's modulus.

Answer We assume that the rod is held by a clamp or vice at one end. Then the force F is applied at the other end, parallel to the length of the rod. Then the stress on the rod is given by

$$\begin{aligned} \text{Stress} &= \frac{F}{A} = \frac{F}{\pi r^2} \\ &= \frac{100 \times 10^3 \text{ N}}{3.14 \times (10^{-2} \text{ m})^2} \\ &= 3.18 \times 10^8 \text{ N m}^{-2} \end{aligned}$$

The elongation,

$$\begin{aligned} \Delta L &= \frac{(F/A)L}{Y} \\ &= \frac{(3.18 \times 10^8 \text{ N m}^{-2})(1 \text{ m})}{2 \times 10^{11} \text{ N m}^{-2}} \\ &= 1.59 \times 10^{-3} \text{ m} \\ &= 1.59 \text{ mm} \end{aligned}$$

The strain is given by

$$\begin{aligned} \text{Strain} &= \Delta L/L \\ &= (1.59 \times 10^{-3} \text{ m})/(1 \text{ m}) \\ &= 1.59 \times 10^{-3} \\ &= 0.159 \% \end{aligned}$$

Table 9.1 Elastic properties of some materials of interest

Substance	Density ρ (kg m^{-3})	Young's modulus Y (10^9 N m^{-2})	Ultimate strength, S_u (10^6 N m^{-2})	Yield strength S_y (10^6 N m^{-2})
Aluminium	2710	70	110	95
Copper	8890	120	400	200
Iron (wrought)	7800-7900	190	330	170
Steel	7860	200	400	250
Glass	2190	65	50	-
Concrete	2320	30	40	-
Wood	525	13	50	-
Bone	1900	9	170	-
Polystyrene	1050	3	48	-

► **Example 9.1** A structural steel rod has a radius of 10 mm and a length of 1m. A 100 kN force F stretches it along its length. Calculate (a) the stress, (b) elongation, and (c) strain on the rod. Given that the Young's modulus, Y , of the structural steel is $2.0 \times 10^{11} \text{ N m}^{-2}$.

9.6.2 Shearing

Elongation is not the only kind of deformation that a solid can experience. As shown in Fig. 9.10 (b), a pair of forces applied perpendicular to the axis, much as we might deform a pack of cards, deforms a cylinder. Similarly, Fig. 9.15 shows a solid with faces that are initially

rectangular. When equal and opposite horizontal forces are applied parallel to the upper and lower faces, the solid deforms so that the upper face has moved sideways with respect to the lower. The horizontal displacement Δl of the upper face is perpendicular to the vertical height l . This type of deformation is called **shear** and the corresponding stress is the **shearing stress**. This type of stress is possible only in solids. It is important to note that in this type of deformation the area of any of the faces is not altered.

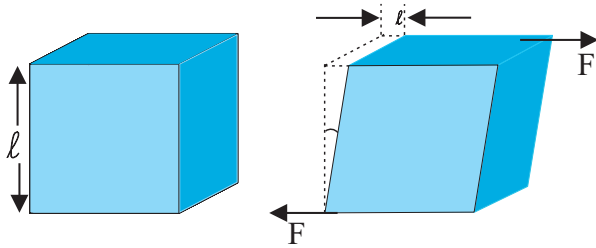


Fig. 9.15 A shear stress twists the cube without altering the area of any of its face

The strain $\Delta l/l$ is equal to $\tan \theta$, where θ is the shear angle as shown in Fig. 9.15. The stress is the tangential force F divided by the area A of the horizontal face of the solid. If Hooke's law is obeyed,

$$\begin{aligned} \frac{F}{A} &= G \frac{\Delta l}{l} \\ &= G \tan \theta \end{aligned} \quad (9.7)$$

The corresponding modulus, which is given by G , is called the **shearing modulus**. Like Young's modulus, the shear modulus is also a force per unit area and therefore is measured in pascals and its SI unit is N m^{-2} .

A common way of determining the shear modulus is to twist a wire or a rod of the material by applying a known force as shown in Fig. 9.16 and measuring the angle of twist.

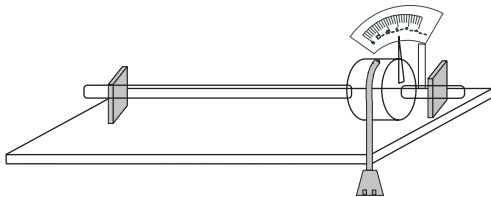


Fig. 9.16 A set up for measuring the shear modulus of a material

It can be shown that a rod of length l and radius R , rigidly fixed at one end, when twisted by applying a torque T at the other end rotates through an angle θ given by

$$\theta = \frac{2Tl}{\pi R^4 G}$$

or

$$G = \frac{2Tl}{\pi R^4 \theta} \quad (9.8)$$

The proof Eq. (9.8) is beyond the scope of this book.

Shearing stress plays an important role in the buckling of shafts that rotate under load and in bone fractures. The shearing moduli of a few common materials are given in Table 9.2.

Table 9.2 Shear modulus G of some common materials

Substance	G (10^9 N m^{-2})
Aluminium	25
Copper	40
Iron	50
Steel	80
Glass	30
Wood	10

Example 9.2 A square lead slab of side 50 cm and thickness 10 cm is subject to a shearing force (on its narrow face) of $9.0 \times 10^4 \text{ N}$. The lower edge is riveted to the floor. How much the upper edge displaced if the shear modulus of lead is $5.6 \times 10^9 \text{ Pa}$?

Answer The lead slab is fixed as shown below (Fig. 9.17). The force is applied parallel to the narrow face as shown. The area of the face parallel to which this force is applied is

$$\begin{aligned} A &= 50 \text{ cm} \times 10 \text{ cm} \\ &= 0.5 \text{ m} \times 0.1 \text{ m} \\ &= 0.05 \text{ m}^2 \end{aligned}$$

Therefore, the stress applied is

$$\begin{aligned} &= \frac{9.0 \times 10^4 \text{ N}}{0.05 \text{ m}^2} \\ &= 1.80 \times 10^6 \text{ N m}^{-2} \end{aligned}$$

$$\text{Strain} = \frac{\Delta l}{l} = \frac{\text{Stress}}{G}$$

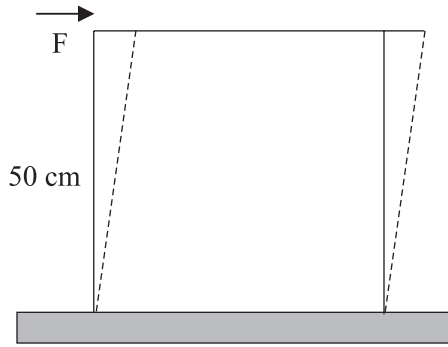


Fig. 9.17

Therefore the displacement,

$$\begin{aligned}
 \Delta l &= \frac{\text{Stress}}{G} \times l \\
 &= \frac{1.8 \times 10^6 \text{ N m}^{-2}}{5.6 \times 10^9 \text{ Pa}} \times 0.5 \text{ m} \\
 &= \frac{1.8 \times 10^6 \text{ N m}^{-2}}{5.6 \times 10^9 \text{ N m}^{-2}} \times 0.5 \text{ m} \quad (1 \text{ Pa} = 1 \text{ N m}^{-2}) \\
 &= 1.6 \times 10^{-4} \text{ m} \\
 &= 0.16 \text{ mm}
 \end{aligned}$$

9.6.3 Hydraulic Stress, Bulk Modulus

In Fig. 9.10(c), the stress equals the fluid pressure p on the object. Pressure is force per unit area acting on the surface of a system, the force being everywhere perpendicular to the surface. In case of uniform pressure, the force per unit area everywhere is the same. Stress also has dimensions of force per unit area, but it does not always act normal to the surface; tensile stress is applied normal to the surface whereas the shearing stress is applied parallel to the surface. Further, unlike the case of pressure, the force per unit area in stress may be different in magnitude on different surfaces. Pressure produces a particular type of stress, which changes only the **volume** of the substance and not its shape. If ΔV is the magnitude of the change in volume and V is the original volume of the substance, then the volume strain is

$$\epsilon_v = \Delta V/V \quad (9.9)$$

The object is under hydraulic compression, and the pressure p is the measure of the **hydraulic stress**. Since the stress is proportional to strain,

we have

$$p = B \frac{\Delta V}{V} \quad (9.10)$$

The constant of proportionality B , is called the **bulk modulus** of the material; its SI unit is N m^{-2} .

For a stable system, hydraulic stress always reduces the volume. Hydraulic stress is exhibited by all the three states of matter, namely the solids, liquids and gas. Bulk modulus of some substances of common interest are given in Table 9.3. It may be noted that the solids are least compressible whereas the gases are most compressible. The bulk modulus of the solids are all in the range of 10^{11} N m^{-2} , and are about 50 times larger than that of water. The incompressibility of the solids is primarily due to their rigid atomic lattice or the tight coupling between the neighbouring atoms. The molecules in liquids and gases are less tightly coupled to their neighbours.

Table 9.3 Bulk modulus of some solids, liquids and gases.

Substance	B (10^9 N m^{-2})
Solids	
Aluminium	70
Copper	120
Iron	80
Steel	158
Glass	36
Liquids	
Ethanol	0.9
Mercury	25
Water	2.2
Gases	
Air (at STP)	1.0×10^{-4}

Example 9.3 The average depth of Indian Ocean is about 3000 m. Calculate the fractional compression, $\Delta V/V$, of water at the bottom of the ocean, given that the bulk modulus of water is $2.2 \times 10^9 \text{ N m}^{-2}$.

Answer The pressure exerted by a column of water of height 3000 m is

$$= 3000 \text{ m} \times 1000 \text{ kg m}^{-3} \times 10 \text{ m s}^{-2} \quad (h \rho g)$$

$$= 3 \times 10^7 \text{ kg m}^{-1} \text{ s}^{-2}$$

$$= 3 \times 10^7 \text{ N m}^{-2} \quad (1 \text{ N} = 1 \text{ kg m s}^{-2})$$

Fractional compression, $\frac{\Delta V}{V}$, for is

$$\begin{aligned}\frac{\Delta V}{V} &= \frac{\text{stress}}{B} = \frac{3 \times 10^7 \text{ N m}^{-2}}{2.2 \times 10^9 \text{ N m}^{-2}} \\ &= 1.36 \times 10^{-2} \\ &= 1.36 \%\end{aligned}$$

9.7 APPLICATIONS OF ELASTIC BEHAVIOUR OF MATERIALS

The elastic behaviour of materials plays an important role in everyday life. All engineering designs require precise knowledge of the elastic behaviour of building materials. For example while designing a building, the structural details of the columns, beams and supports require knowledge of strength of materials used. Have you ever thought why the beams used in construction of bridges, as supports etc have a cross-section of the type I? Why does a heap of sand or a hill have a pyramidal shape? Answers to these questions are complex and constitute the full discipline of structural engineering. To illustrate our point we will take up only a few examples.

Cranes used for lifting and moving heavy loads from one place to another have a thick metal rope to which the load is attached. The rope is pulled up using pulleys and motors. Suppose we want to construct a crane, which has a lifting capacity of 10 metric tons. How thick should the steel rope be? We obviously want that the load does not deform the rope permanently; therefore, the extension should not exceed the elastic limit. From Table 9.1, we find that steel has yield strength of $300 \times 10^6 \text{ N m}^{-2}$. Therefore, the area of cross-section of the rope should at least be

$$\begin{aligned}A &\geq W/S_y = Mg/S_y \\ &= \frac{10^4 \text{ kg} \times 10 \text{ m s}^{-2}}{300 \times 10^6 \text{ N m}^{-2}} \\ &= \frac{10^5 \text{ N}}{3 \times 10^8 \text{ N m}^{-2}} \\ &= 3.3 \times 10^{-4} \text{ m}^2\end{aligned}\quad (9.11)$$

corresponding to a radius of about 1 cm for a rope of circular cross-section. Allowing for some safety factor (~ 10) the recommended radius would be about 10 cm. A single wire of this radius

would practically be a rigid rod, so the ropes are always made of a number of thin wires braided together, like in pigtails, for ease in manufacture, flexibility and strength.

A bridge has to be designed such that it can withstand the load of the flowing traffic, the force of winds and its own weight. Similarly, in the design of buildings use of beams and columns is very common. In both these problems the bending of beams under a load is of prime importance. The beam should not bend too much or break. Let us, therefore, consider the case of a beam loaded at the centre and supported near its ends as shown in Fig. 9.18. A bar of length l , breadth b , and thickness d when loaded at the centre by a load W sags by an amount given by

$$\delta = \frac{Wl^3}{4bd^3Y} \quad (9.12)$$

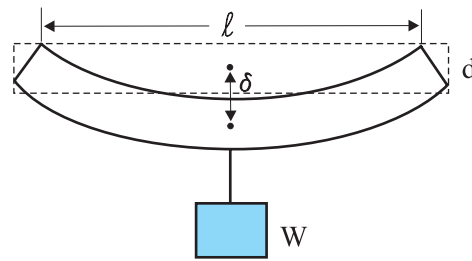


Fig. 9.18 A beam supported at the ends and loaded at the centre

This relation can be derived using what you have already learnt and a little calculus. From Eq. (9.12), we see that to reduce the bending for a given load, one should use a material with a large Young's modulus Y . For a given material, increasing the thickness d rather than the breadth b is more effective in reducing the bending since δ is proportional to d^{-3} and only to b^{-1} . However a deep bar may have a tendency to buckle as shown in Fig. 9.19 which shows different sectional shapes of a bar. To avoid this, a common compromise is the cross-sectional shape shown in Fig. 9.19(c). This section provides a large load bearing surface and enough depth to prevent bending. This shape reduces the weight of the beam without sacrificing the strength and hence reduces the cost.

Use of pillars or columns is also very common in buildings and bridges.

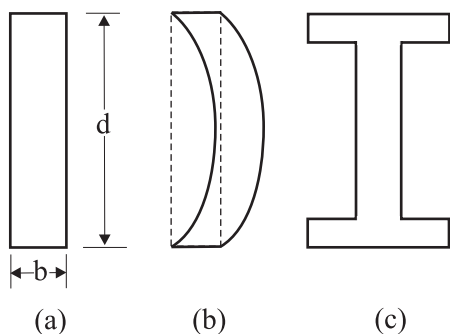


Fig. 9.19 Different cross-sectional shapes of a beam. (a) Rectangular section of a bar; (b) A thin bar and how it can buckle; (c) Commonly used section for a load bearing bars

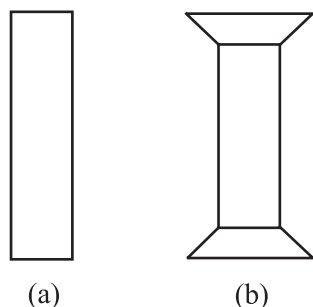


Fig. 9.20 Pillars or columns: (a) a pillar with rounded ends, (b) Pillar with distributed ends

A pillar with rounded ends as shown in Fig. 9.20 (a) supports less load than a distributed shape at the ends [Fig. 9.20 (b)].

An answer to the question why the maximum height of a mountain on earth is ~ 10 km can also be provided by considering the elastic properties of rocks. A mountain base is not under uniform compression; this provides some shearing stress to the rocks under which they can flow. The stress due to all the material on the top should be less than the critical shearing stress at which the rocks flow.

At the bottom of a mountain of height h the force per unit area due to the weight of the mountain is $h\rho g$ where ρ is the density of the material of the mountain and g is the acceleration due to gravity. The material at the bottom experiences this force in the vertical direction, and the sides of the mountain are free. Therefore this is not a case of pressure or bulk compression. There is a shear component, approximately $h\rho g$ itself. Now the elastic limit for a typical rock is $30 \times 10^7 \text{ N m}^{-2}$, equating this to $h\rho g$ gives

$$h\rho g = 30 \times 10^7 \text{ N m}^{-2}$$

$$\text{Or } h = \frac{30 \times 10^7 \text{ N m}^{-2}}{\rho g}$$

$$= \frac{30 \times 10^7 \text{ N m}^{-2}}{3 \times 10^3 \text{ kg m}^{-3} \times 10 \text{ m s}^{-2}}$$

$$= 10 \text{ km}$$

which is more than the height of Mt. Everest.

SUMMARY

1. All matter consists of extremely small discrete bits (atoms or molecules) that are in incessant motion.
2. Evidence for atoms comes from the empirical laws of chemical combination and successful predictions of kinetic theory. Brownian motion strikingly confirms the reality of atoms and its quantitative treatment yields the value of Avogadro's constant.
3. Interatomic potential energy has both an attractive and repulsive part. As the distance R decreases, the attractive force increases. At a certain distance R_0 , potential energy is minimum. For R equal to R_0 , the force is repulsive.
4. The liquid phase results from the attractive force between atoms or molecules. The repulsive part of the force plays a crucial role in the transition to the solid phase.
5. A solid is a large conglomeration of atoms or molecules ($\sim 10^{23}$). They have definite shape and size. The solids we come across in our daily life could be classified in following groups :
 - a) crystalline solids
 - b) semi-crystalline solids and
 - c) amorphous solids
6. Crystals are formed by a regular repetition of identical building blocks. The elementary building blocks are atoms or group of atoms. The crystals are a three-dimensional periodic array of atoms called the lattice. In a lattice, each atom has a well-defined

equilibrium distance from its nearest neighbours. Inter-atomic forces hold the atoms together.

7. Atoms in amorphous or glassy solids are arranged in a random manner as in a liquid. But unlike liquids where the molecules move relative to each other, here the atomic or molecular locations are fixed with respect one another. As a consequence, these solids like the crystalline solids also possess definite shape and size.
8. The crystalline and amorphous phases co-exist in semi crystalline solids. Some of the polymers provide examples of such solids.
9. Stress is the restoring force per unit area and strain is the unit deformation produced by the applied stress. In general there are three types of stresses (a) tensile stress (associated with stretching), (b) shearing stress, and (c) compressional or hydraulic stress.
10. For small deformations **stress $\propto$ strain**. This is known as Hooke's law. The constant of proportionality is called a **modulus of elasticity**. Three elastic moduli are used to describe the elastic behaviour (deformation) of objects as they respond to deforming forces that act on them.

A class of solids called elastomers does not obey Hooke's law.

11. When an object is under tension or compression, the Hooke's law takes the form

$$F/A = Y\Delta L/L$$

Where $\Delta L/L$ is the tensile or compressive strain of the object, F is the magnitude of the applied force causing the strain, A is the cross-sectional area over which F is applied (perpendicular to A) and Y is the Young's modulus for the object. The stress is F/A .

12. A pair of forces when applied parallel to the upper and lower faces, the solid deforms so that the upper face moves sideways with respect to the lower. The horizontal displacement Δl of the upper face is perpendicular to the vertical height l . This type of deformation is called **shear** and the corresponding stress is the **shearing stress**. This type of stress is possible only in solids.

In this kind of deformation the Hooke's law takes the form

$$F/A = G \Delta l/l$$

where Δl is the displacement of one end of object in the direction of the applied force F , and G is the shear modulus. The stress is F/A .

13. When an object undergoes **hydraulic compression** due to a stress exerted by a surrounding fluid, the Hooke's law takes the form

$$p = B (\Delta V/V),$$

where p is the pressure (**hydraulic stress**) on the object due to the fluid, $\Delta V/V$ (the strain) is the absolute fractional change in the object's volume due to that pressure, and B is the **bulk modulus** of the object.

Physical Quantity	Symbol	Dimensions	Unit	Remarks
Stress	σ	$[ML^{-1}T^{-2}]$	$N\ m^{-2}$	Stress strain (Hooke's law)
Strain	ε	Dimensionless	No units	
Young's modulus	Y	$[ML^{-1}T^{-2}]$	$N\ m^{-2}$	Applies to change in length (relevant for solids)
Shear modulus	G	$[ML^{-1}T^{-2}]$	$N\ m^{-2}$	Applies to change in shape (relevant for solids)
Bulk modulus	B	$[ML^{-1}T^{-2}]$	$N\ m^{-2}$	Applies to volumetric strain (relevant to solid, liquid and gases)

POINTS TO PONDER

1. The linear relation between stress and strain (Hooke's Law) is not valid for large values of strains.
2. Only a single force acting on a body initially at rest is bound to cause its translational motion. Two equal and opposite forces acting on a body along the same line result in its deformation without any translational motion of its centre of mass. For example, for a wire under tension by a weight suspended at one end, the two forces are (i) force on the wire due to the weight and (ii) the force by the ceiling on the other end of the wire. (iii) Stress is related to *internal restoring force* that comes into play due to any deformation produced by the external forces. For a wire under tension by two equal and opposite forces at its ends, each of magnitude F , the stress is F/A and not $2F/A$ where A is the area of cross section of the wire.
3. The Young's modulus and shear modulus are relevant only for solids since only solids have lengths and shapes.
4. Bulk modulus is relevant for solids, liquid and gases. It refers to the change in volume when every part of the body is under the **same** stress so that the shape of the body remains unchanged. Thus, any change in volume is not directly related to the bulk modulus. For example, for a wire under longitudinal strain, the lateral dimensions (radius of cross section) will undergo small change, which is described by another elastic constant of the material (called *Poisson ratio*).
5. In general, a deforming force in one direction can produce strains in other directions also. The proportionality between stress and strain in such situations cannot be described by just only one elastic constant. (You will learn in more advance courses that elastic co-efficients are '*tensors*').

EXERCISES

- 9.1** The black oxide of copper is prepared in four different ways and the following observations noted:

Copper (in g)	Copper oxide (in g)
5.015	6.284
8.286	10.357
18.443	23.169
12.011	15.051

What do you infer from this data ?

- 9.2** Given below are data on masses of two combining substances yielding more than one compound. Try to discover a striking feature hidden in these data.

Carbon (parts by mass)	Oxygen (parts by mass)	Compound
464.9	626.7	Carbon monoxide
789.3	2086.2	Carbon dioxide
Carbon	Hydrogen	Compound
334.6	111.2	Methane
854.5	146.4	Ethylene
Element	Sulphur	Compound
150.0	75.6	I
266.4	68.8	II

- 9.3** Given the chemical formula for methane to be CH_4 , obtain the ratio of mass of a carbon atom to the mass of a hydrogen atom from the data in Exercise 9.2, and predict some possible chemical formulae for ethylene.
- 9.4** The following table gives observations on two gaseous reactions. In each set, the temperature and pressure conditions are kept fixed.

A. Nitrogen gas (cm^3)	Hydrogen gas (cm^3)	Ammonia gas (cm^3)
623.3	1830.0	1253.4
349.0	1051.2	685.6
84.7	251.9	170.7
B. Hydrogen gas (cm^3)	Oxygen gas (cm^3)	Water vapour (cm^3)
307.9	156.6	309.1
435.9	217.8	432.6
851.1	473.1	856.0

What do you infer from these data? Can you understand the law you have inferred empirically on the basis of the atomic picture and Avogadro's hypothesis?

- 9.5** (a) Avogadro's hypothesis is: "Equal volumes of all gases under the same conditions of temperature and pressure have the same number of molecules". Suppose we replaced 'molecules' by 'atoms' in this hypothesis. (By atoms we refer to indivisible entities in chemical reactions, unlike molecules, which can split into atoms). Would the modified hypothesis be correct? Would it explain the data in 9.4?
- (b) What is the atomicity of nitrogen, oxygen, and hydrogen suggested by the data in 9.4? (atomicity is the number of atoms in a molecule).
- 9.6** A drop of olive oil of 1 mm diameter when transferred lightly over water dusted with lycopodium powder spreads out into a circular thin film of diameter 28.1 cm. Estimate the size of a molecule of the oil assuming that the film is only one molecule thick.
- 9.7** A steel wire of length 4.7 m and cross-section $3.0 \times 10^{-5} \text{ m}^2$ stretches by the same amount as a copper wire of length 3.5 m and cross-section $4.0 \times 10^{-5} \text{ m}^2$ under a given load. What is the ratio of the Young's modulus of steel to that of copper?
- 9.8** Figure 9.21 shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?

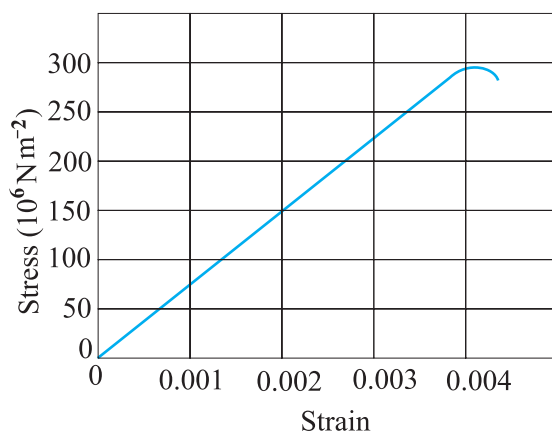


Fig. 9.21

9.9 The stress-strain graphs for materials A and B are shown in Fig. 9.22

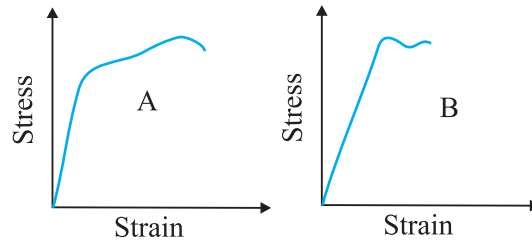


Fig. 9.22

The graphs are drawn to the same scale.

- (a) Which of the material has greater Young's modulus ?
 (b) Which material is more ductile ?
 (c) Which is more brittle ?
 (d) Which of the two is stronger material ?
- 9.10** Two different types of rubber are found to have the stress-strain curves as shown in Fig. 9.23.

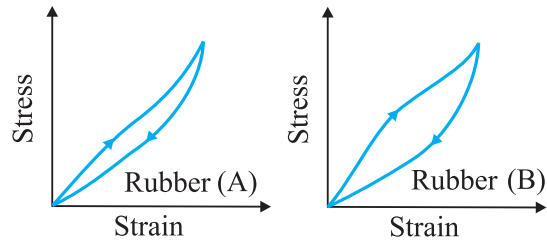


Fig. 9.23

- (a) In which significant ways do these curves differ from the stress-strain curve of a metal wire shown in Fig. 9.12 ?
 (b) A heavy machine is to be installed in a factory. To absorb vibrations of the machine, a block of rubber is placed between the machinery and the floor. Which of the two rubbers A and B would you prefer to use for this purpose ? Why ?
 (c) Which of the two rubber materials would you choose for a car tire ?
- 9.11** Read each of the statements below carefully and state, with reasons, if it is true or false.
- (a) The modulus of elasticity of rubber is greater than that of steel
 (b) The stretching of a coil is determined by its shear modulus
 (c) When a material is under tensile stress, the restoring forces are caused by inter-atomic attraction. While under compressional stress the restoring force is due to inter-atomic repulsion
 (d) A piece of rubber under an ordinary stress can display 1000% strain; yet when unloaded it returns to its original length. This shows that the restoring forces in a rubber piece are strictly conservative
 (e) Elastic forces are strictly conservative when Hooke's law is obeyed

- 9.12** Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in Fig. 9.24. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Young's modulus of steel is 2.0×10^{11} Pa and that of brass is 0.91×10^{11} Pa. Compute the elongations of steel and brass wires. ($1 \text{ Pa} = 1 \text{ N m}^{-2}$)

- 9.13** The edges of an aluminium cube are 10 cm long. On face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face ? ($1 \text{ Pa} = 1 \text{ N m}^{-2}$)

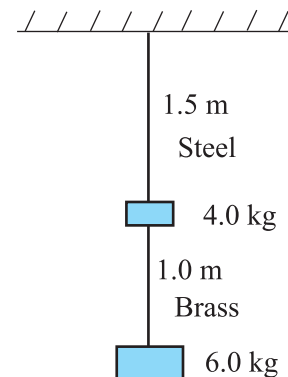


Fig. 9.24

- 9.14** A silica glass rod has a diameter of 1 cm and is 10 cm long. Using the data from Table 9.1, estimate the largest mass that can be hung from it without breaking it.
- 9.15** Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm ($1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large. ($1 \text{ Pa} = 1 \text{ N m}^{-2}$)
- 9.16** What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is $1.03 \times 10^3 \text{ kg m}^{-3}$? (Compressibility of water is $45.8 \times 10^{-11} \text{ Pa}^{-1}$; $1 \text{ Pa} = 1 \text{ N m}^{-2}$)

Additional Exercises

- 9.17** You will appreciate from Exercise 9.3 that data on chemical combinations can at best yield ratios of atomic masses of different elements. This is why atomic masses are defined with respect to some 'reference atom'. In early chemistry, hydrogen was chosen to be the reference and assigned a unit mass. In the so called chemical scale, oxygen was assigned a mass of 16 units. By international agreement, the unified atomic mass unit (u) is defined to be such that the isotope ^{12}C has a mass exactly equal to 12 u. The difference between these different scales is slight but important for precision measurements.
- The mass of ^{137}Cs atom is 136.90707 u on the unified atomic scale. The mass of ^1H is 1.0078252 u on the same scale. What is the mass of ^{137}Cs atom with respect to the hydrogen reference scale?
- 9.18** Why are atomic masses (on say the unified scale) not exact integers? Why is there a slight difference in the atomic mass of an element on different scales?
- 9.19** (a) If data on chemical combination give only relative atomic masses, how does one know the absolute mass of an atom of say oxygen?
 (b) Express 1 u in terms of kg, given that the experimental value of Avogadro's number is $N_A = 6.022045 \times 10^{23} \text{ mol}^{-1}$
- 9.20** Figure 9.1 in the text gives a plot of interatomic potential energy as function of the distance between two hydrogen atoms. Answer the following:
- (a) if interatomic forces are electrical in nature, why do they not obey Coulomb's law; i.e. why do they fall off as $1/r^7$ for a large distance (instead of as $1/r^2$ expected on the basis of Coulomb's law).
 (b) what is the physical origin of the attractive and repulsive parts of interatomic force?
 (c) if the potential energy minimum is at $r = R_0 = 0.74 \text{ \AA}$, is the force attractive or repulsive at $r = 0.5 \text{ \AA}$, $1.9 \text{ \AA}$ and ∞ ?
- 9.21** How much energy is needed to dissociate 0.05% of hydrogen gas occupying a volume of 5.6 litres under STP? The binding energy of a hydrogen molecule is 4.75 eV.
- 9.22** The interatomic separation in a neutral H_2 molecule is $0.74 \text{ \AA}$ and the binding energy is 4.75 eV. If an electron of the molecule is removed, the resulting molecular ion H_2^+ has a binding energy of 2.8 eV. Would you expect the separation between two protons in H_2^+ greater or less than $0.74 \text{ \AA}$?
- 9.23** Explain (a) An energy of about 1.3 eV is needed to create free Na^+ and Cl^- ions. [By this we mean that the $\text{Na}^+ \text{Cl}^-$ configuration at large distances has energy 1.3 eV higher than neutral Na and Cl atoms at large distances]. In spite of this initial investment the sodium chloride molecule prefers ionic binding. Why?
 (b) A related question is why does H_2 prefer a covalent binding? Why does it not prefer ionic binding $\text{H}^+ \text{H}^-$ like $\text{Na}^+ \text{Cl}^-$?
- 9.24** The interatomic separation in an O_2 molecule is $1.2 \text{ \AA}$, and its binding energy is about 4.4 eV. The intermolecular potential energy between two oxygen molecules has a minimum at $2.9 \text{ \AA}$. The two potential energy curves have roughly similar shapes. Would you expect the minimum of intermolecular potential energy in oxygen to be greater or less than 4.4 eV? Check your expectation with the answer given in the text.
- 9.25** In which principal aspects are intermolecular forces different from interatomic forces? In which aspects are they similar?

- 9.26** Estimate the average intermolecular binding energy per molecule of (i) mercury, (ii) water if their latent heats of vaporisation are $2.72 \times 10^5 \text{ J kg}^{-1}$ and $2.26 \times 10^6 \text{ J kg}^{-1}$ respectively. Is it correct to think that this average binding energy is the same, as that required pulling two molecules of the substance apart from one another, i.e. the negative of the minimum of potential energy between two molecules ?
- 9.27** Answer the following:
- If intermolecular potential energy has a minimum at some separation $r = r_0$, what is it that prevents all molecules of the given substance from collapsing to the condensed state where every pair has separation equal to r_0 ?
 - Intermolecular attraction is the crucial thing for gas to liquid transition, while a repulsive potential energy at short distances is crucial for liquid to solid transition. Explain this important qualitative insight into the nature of phase transitions (Read section 9.3).
 - Give an example of an unusual phase of matter in nature, which arises due to the highly non-spherical shape of its molecules.
- 9.28** Four identical hollow cylindrical columns of steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. The Young's modulus of steel is $2.0 \times 10^{11} \text{ Pa}$ ($1 \text{ Pa} = 1 \text{ N m}^{-2}$).
- 9.29** Anvils made of single crystals of diamond, with the shape as shown in Fig. 9.25, are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.5 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil ?

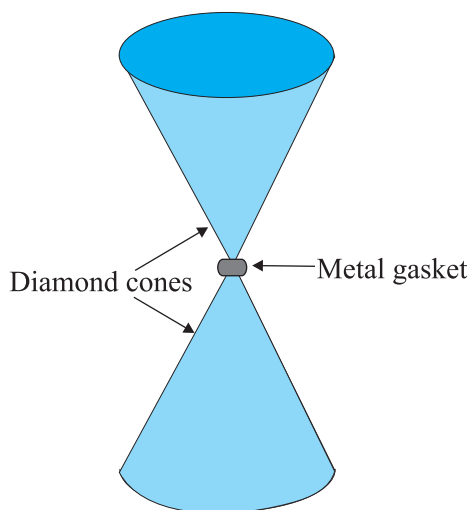


Fig. 9.25

- 9.30** A composite wire of uniform diameter 3.0 mm consists of a copper wire of length 2.2 m and a steel wire of length 1.6 m stretches under a load by 0.7 mm. Calculate the load, given that the Young's modulus for copper is $1.1 \times 10^{11} \text{ Pa}$ and that for steel is $2.0 \times 10^{11} \text{ Pa}$ ($1 \text{ Pa} = 1 \text{ N m}^{-2}$).
- 9.31** (a) You have discovered in Exercises 9.1 and 9.2 the 'law of constant proportions' and the 'law of multiple proportions'. Which of these two laws in your view suggests the atomic hypothesis ? Explain carefully.
- (b) Would you regard Dalton's atomic hypothesis a fundamental improvement over the atomic views of ancient Indians and Greeks ? Justify your answer.